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Dynamic stress concentration around elliptic cavities in saturated poroelastic soil under harmonic plane waves

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Abstract

Potential function and complex function in the elliptic coordinate system are employed to solve the problem of scattering harmonic plane waves by multiple elliptic cavities in water saturated soil medium. The steady state Biot's dynamic equations of poroelasticity are uncoupled into Helmholtz equations via given potentials. The stresses and pore water pressures are obtained by using complex functions in elliptic coordinates with certain boundary conditions. Finally, the dynamic stresses for the case of two interacting elliptic cavities are obtained and discussed in details via a numerical example.

Keywords: Saturated soil; Elliptic cavities; Plane wave; Complex function; Helmholtz equation

1. Introduction

Diffraction of elastic waves by obstacles such as cavities, cracks, and inserts has been investigated by many researchers. Investigation on dynamic stress concentrations in solids is very significant in the study of dynamic strength of materials and in the design of underground structures subject to ground blasting waves. Scattering elastic waves by a single cavity has been treated by many researchers. Zitron (1967) dealt with the multiple scattering of plane elastic waves by two arbitrary cylinders in a homogeneous medium. Gamer (1977) used wave function expansion method to study dynamic stress concentration factor at the surface of a semi-circular cavity in an elastic half-space excited by plane harmonic wave. Liu et al. (1981) presented an analytical method for cylindrical canyon of arbitrary shape and for incident plane wave by using complex

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functions and mapping techniques. Sancar and Pao (1981) gave solutions for the scattering of plane harmonic pressure waves by two cylindrical cavities in an elastic solid by using the eigenfunction expansion methods. Datta et al. (1984) studied the dynamic stresses and displacements around cylindrical cavities of various shapes in an elastic medium by employing a combined approach of finite element method (FEM) and the method of eigenfunction expansions. Several other researchers also studied the scattering of elastic wave by cavity embedded in a poroelastic medium. Mei and Foda (1981) and Mei et al. (1984) introduced a simple method for a circular cavity of arbitrary radius in a poroelastic medium and for both P and SV incident waves by using the boundary layer approximation. Norris (1985) obtained the solution for a point load in an unbounded fluid–saturated porous solid. Zimmerman and Stern (1993) studied the problem of wave diffraction by a spherical cavity in an infinite poroelastic soil medium by using the boundary element method (BEM). Degrande et al. (1998) studied harmonic and transient wave propagation in multilayered saturated and unsaturated porous medium. Kattis et al. (2003) used BEM to solve the problem of incident harmonic P and SV plane waves by tunnels in an infinite poroelastic saturated soil.

It is obviously that FEM and BEM have been successfully applied to solve wave diffraction problem in various dynamic poroelastic medium. In this paper, the complex variable method in multi-polar coordinate system is employed to solve the scatter of plane wave by elliptic cavities embedded in an infinite saturated soil. The proposed method can be applied to solve the problem of scattering harmonic plane wave through cavities in saturated soil with less computational effort. The behavior of the saturated soil is governed by Biot's consolidation theory. Then, these equations are decoupled via introducing the potential functions and reduced to Helmholtz equations that the potentials satisfy. Applying the boundary conditions of the solid matrix and the fluid, the solutions of dynamic stress concentration and pore pressure concentration of plane wave by two elliptic cavities in the saturated soil are presented.

2. Governing equations and general solutions

Based on Biot's theory for a two-phased material, the constitutive relations for saturated soil are expressed as (Biot, 1941, 1956, 1962)

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}e - \alpha\delta_{ij}p_{\mathrm{f}} \quad (i, j = x, y) \tag{1}$$

$$p_{\mathrm{f}} = -\alpha Me + M\vartheta \tag{2}$$

$$e = u_{i,i}, \quad \vartheta = -w_{i,i} \tag{3}$$

where σ_{ij} is the total stress components of the bulk material; ε_{ij} and e are the strain component and dilatation of the solid matrix, respectively; λ and μ are Lamé's constants; δ_{ij} is Kronecker delta; ϑ is the variation of fluid content per unit reference volume; α and M are Biot's parameters, respectively; p_{f} is the pore pressure.

The equations of motion for the poroelastic medium can be expressed in terms of displacements, u_i and w_i, as

$$\mu u_{i,jj} + (\lambda + \alpha^2 M + \mu)u_{j,ji} + \alpha M w_{j,ji} = \rho\ddot{u}_i + \rho_{\mathrm{f}}\ddot{w}_i \tag{4}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_{\mathrm{f}}\ddot{u}_i + \frac{\rho_{\mathrm{f}}}{n}\ddot{w}_i + \frac{\eta}{k}\dot{w}_i \tag{5}$$

where u_i is the solid displacement; w_i is the fluid displacement; ρ and ρ_{f} are the mass densities of the bulk material and the pore fluid, respectively; $\rho = (1 - n)\rho_{\mathrm{s}} + \rho_{\mathrm{f}}$, ρ_{s} is the soil density; n is porosity; k is the permeability; η is the fluid viscosity; over-dots denote the derivatives of field variables with respect to time t.

Three potentials are adopted: the scalar potentials φ_f and φ_s to represent the fast and slow compressible waves, respectively and the vector potential ψ_k to represent the shear wave. These potentials relate to u_i and p_f are expressed as follows (Zimmerman and Stern, 1993):

$$u_i = \varphi_{,i} + e_{ijk}\psi_{k,j} = \varphi_{f,i} + \varphi_{s,i} + e_{ijk}\psi_{k,j} \tag{6}$$

$$p_f = A_f\varphi_{f,ii} + A_s\varphi_{s,ii} \tag{7}$$

where A_f and A_s are amplitude ratios for fast and slow wave, respectively; and e_{ijk} is tensor transformation.

When considering the time harmonic vibration of frequency ϖ by the term $e^{-i\varpi t}$, where $i = \sqrt{-1}$, for brevity, the term $e^{-i\varpi t}$ is suppressed henceforth from all expressions in the sequel. Substituting Eq. (2) and Eqs. (6) and (7) into Eq. (4), the following formula is obtained:

$$[(\lambda + 2\mu - \beta_2 A_f)\varphi_{f,jj} + \beta_3\varphi_f]_{,i} + [(\lambda + 2\mu - \beta_2 A_s)\varphi_{s,jj} + \beta_3\varphi_s]_{,i} + e_{iml}[\mu\psi_{l,jj} + \beta_3\psi_l]_{,m} = 0 \tag{8}$$

In order to satisfy Eq. (8), the expressions in braces should be equal to zero independently. Thus, Eq. (8) can be written in the following form:

$$(\lambda + 2\mu - \beta_2 A_f)\varphi_{f,jj} + \beta_3\varphi_f = 0 \tag{9}$$

$$(\lambda + 2\mu - \beta_2 A_s)\varphi_{s,jj} + \beta_3\varphi_s = 0 \tag{10}$$

$$\mu\psi_{i,jj} + \beta_3\psi_i = 0 \tag{11}$$

where

$$\beta_1 = -\frac{\rho_f\varpi^2}{n} - \frac{i\eta\varpi}{k}; \quad \beta_2 = \alpha + \frac{\rho_f\varpi^2}{\beta_1}; \quad \beta_3 = \rho\varpi^2 + \frac{\rho_f^2\varpi^4}{\beta_1} \tag{12}$$

Substituting Eqs. (2) and (3) into Eq. (5), one obtains

$$p_{f,ii} - \frac{\beta_1}{M}p_f - (\alpha\beta_1 + \rho_f\varpi^2)u_{i,i} = 0 \tag{13}$$

Substituting Eqs. (6) and (7) into Eq. (13), one has

$$[A_f\varphi_{f,ii} + (\beta_5 A_f - \beta_4)\varphi_f]_{,jj} + [A_s\varphi_{s,ii} + (\beta_5 A_s - \beta_4)\varphi_s]_{,jj} = 0 \tag{14}$$

In order to satisfy Eq. (14), the expressions in braces should be equal to zero independently. Thus, one has

$$A_f\varphi_{f,ii} + (\beta_5 A_f - \beta_4)\varphi_f = 0 \tag{15}$$

$$A_s\varphi_{s,ii} + (\beta_5 A_s - \beta_4)\varphi_s = 0 \tag{16}$$

where

$$\beta_4 = \alpha\beta_1 + \rho_f\varpi^2, \quad \beta_5 = -\beta_1/M \tag{17}$$

From Eqs. (9) and (10) and Eqs. (15) and (16), one has

$$A_{f,s}^2 + \frac{\beta_3 - (\lambda + 2\mu)\beta_5 - \beta_2\beta_4}{\beta_2\beta_5}A_{f,s} + \frac{(\lambda + 2\mu)\beta_4}{\beta_2\beta_5} = 0 \tag{18}$$

From Eqs. (9)–(11) and (15) and (16), each component $\varphi_{f,s}$ and ψ must satisfy Helmholtz equation of the following form:

$$\nabla^2\varphi_f + k_f^2\varphi_f = 0 \tag{19}$$

$$\nabla^2\varphi_s + k_s^2\varphi_s = 0 \tag{20}$$

$$\nabla^2\psi + k_t^2\psi = 0 \tag{21}$$

where

$$k_f^2 = \beta_3/(\lambda + 2\mu - \beta_2 A_f) = (\beta_5 A_f - \beta_4)/A_f \tag{22}$$

$$k_s^2 = \beta_3/(\lambda + 2\mu - \beta_2 A_s) = (\beta_5 A_s - \beta_4)/A_s \tag{23}$$

$$k_t^2 = \beta_3/\mu \tag{24}$$

where k_f, k_s, and k_t are the complex wave numbers associated with the fast wave, slow wave, and shear wave, respectively. If selecting $\mathrm{Im}(k_{f,s,t}) > 0$ to give a wave that decays as the wave propagates outward, there is $\mathrm{Re}(k_f) < \mathrm{Re}(k_s)$.

3. Expressions of displacements, stresses, and pore pressures

The expressions of displacements u_i and w_i $(i = x, y)$ and pore pressure p_f can be expressed as

$$u_x = \frac{\partial\varphi_f}{\partial x} + \frac{\partial\varphi_s}{\partial x} + \frac{\partial\psi}{\partial y} \tag{25}$$

$$u_y = \frac{\partial\varphi_f}{\partial y} + \frac{\partial\varphi_s}{\partial y} - \frac{\partial\psi}{\partial x} \tag{26}$$

$$w_x = \eta_1\frac{\partial\varphi_f}{\partial x} + \eta_2\frac{\partial\varphi_s}{\partial x} + \alpha_1\frac{\partial\psi}{\partial y} \tag{27}$$

$$w_y = \eta_1\frac{\partial\varphi_f}{\partial y} + \eta_2\frac{\partial\varphi_s}{\partial y} - \alpha_1\frac{\partial\psi}{\partial x} \tag{28}$$

$$\sigma'_x = \lambda(\nabla^2\varphi_f + \nabla^2\varphi_s) + 2\mu\left(\frac{\partial^2\varphi_f}{\partial x^2} + \frac{\partial^2\varphi_s}{\partial x^2} + \frac{\partial^2\psi}{\partial y\partial x}\right) \tag{29}$$

$$\sigma'_y = \lambda(\nabla^2\varphi_f + \nabla^2\varphi_s) + 2\mu\left(\frac{\partial^2\varphi_f}{\partial y^2} + \frac{\partial^2\varphi_s}{\partial y^2} + \frac{\partial^2\psi}{\partial x\partial y}\right) \tag{30}$$

$$\sigma'_{xy} = \mu\left(2\frac{\partial^2\varphi_f}{\partial y\partial x} + 2\frac{\partial^2\varphi_s}{\partial y\partial x} + \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial x^2}\right) \tag{31}$$

$$p_f = -A_f k_f^2\varphi_f - A_s k_s^2\varphi_s \tag{32}$$

where superscript (′) denotes effective stress and

$$\eta_1 = \alpha_1 - \alpha_2 A_f k_f^2, \quad \eta_2 = \alpha_1 - \alpha_2 A_s k_s^2, \quad \alpha_1 = \frac{\rho_f\varpi^2}{\beta_1}, \quad \alpha_2 = -\frac{1}{\beta_1} \tag{33}$$

If introducing complex variables $z = x + \mathrm{i}y$, $\bar{z} = x - \mathrm{i}y$ and let γ to represent polar coordinate rotary angle, the displacements, stresses, and pore pressures are expressed as:

$$u_{\tilde{x}} + \mathrm{i}u_{\tilde{y}} = 2\frac{\partial}{\partial \bar{z}}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} - \mathrm{i}\psi)\mathrm{e}^{-\mathrm{i}\gamma} \tag{34}$$

$$u_{\tilde{x}} - \mathrm{i}u_{\tilde{y}} = 2\frac{\partial}{\partial z}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} + i\psi)\mathrm{e}^{\mathrm{i}\gamma} \tag{35}$$

$$w_{\tilde{x}} + \mathrm{i}w_{\tilde{y}} = 2\frac{\partial}{\partial \bar{z}}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} - \alpha_1\mathrm{i}\psi)\mathrm{e}^{-\mathrm{i}\gamma} \tag{36}$$

$$w_{\tilde{x}} - \mathrm{i}w_{\tilde{y}} = 2\frac{\partial}{\partial z}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} + \alpha_1\mathrm{i}\psi)\mathrm{e}^{\mathrm{i}\gamma} \tag{37}$$

$$\sigma'_{\tilde{x}} - \mathrm{i}\sigma'_{x\tilde{y}} = -(\lambda+\mu)(k_{\mathrm{f}}^2\varphi_{\mathrm{f}} + k_{\mathrm{s}}^2\varphi_{\mathrm{s}}) + \mu\frac{\partial^2}{\partial z^2}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} + \mathrm{i}\psi)\mathrm{e}^{2\mathrm{i}\gamma} \tag{38}$$

$$\sigma'_{\tilde{x}} + \mathrm{i}\sigma'_{x\tilde{y}} = -(\lambda+\mu)(k_{\mathrm{f}}^2\varphi_{\mathrm{f}} + k_{\mathrm{s}}^2\varphi_{\mathrm{s}}) + \mu\frac{\partial^2}{\partial \bar{z}^2}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} - \mathrm{i}\psi)\mathrm{e}^{-2\mathrm{i}\gamma} \tag{39}$$

As shown in the above equations, pore pressure is not changed in the new coordinate.

4. Solutions of boundary problems

For stress boundary problems, the two cases are considered: permeable boundary and impermeable boundary. For permeable boundary, the pore pressure on the boundary of cavity is zero. For impermeable boundary, the normal displacement of fluid relative to the solid matrix is zero.

Using Eqs. (38) and (39), the stresses on the boundary of cavity can be expressed as

$$\sigma_{\tilde{x}} - \mathrm{i}\sigma_{x\tilde{y}} = \alpha_{\mathrm{f}}\varphi_{\mathrm{f}} + \alpha_{\mathrm{s}}\varphi_{\mathrm{s}} + \frac{\partial^2}{\partial z^2}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} + \mathrm{i}\psi)\mathrm{e}^{2\mathrm{i}\gamma} = f_1 - \mathrm{i}f_2 \tag{40}$$

$$\sigma_{\tilde{x}} + \mathrm{i}\sigma_{x\tilde{y}} = \alpha_{\mathrm{f}}\varphi_{\mathrm{f}} + \alpha_{\mathrm{s}}\varphi_{\mathrm{s}} + \frac{\partial^2}{\partial \bar{z}^2}(\varphi_{\mathrm{f}} + \varphi_{\mathrm{s}} - \mathrm{i}\psi)\mathrm{e}^{-2\mathrm{i}\gamma} = f_1 + \mathrm{i}f_2 \tag{41}$$

where f_1 and f_2 are the normal and tangential total stresses and

$$\alpha_{\mathrm{f}} = \alpha A_{\mathrm{f}}k_{\mathrm{f}}^2 - (\lambda+\mu)k_{\mathrm{f}}^2 \tag{42}$$

$$\alpha_{\mathrm{s}} = \alpha A_{\mathrm{s}}k_{\mathrm{s}}^2 - (\lambda+\mu)k_{\mathrm{s}}^2 \tag{43}$$

For permeable boundary, the pore pressure is zero. Eq. (32) can be written as

$$p_{\mathrm{f}} = -A_{\mathrm{f}}k_{\mathrm{f}}^2\varphi_{\mathrm{f}} - A_{\mathrm{s}}k_{\mathrm{s}}^2\varphi_{\mathrm{s}} = 0 \tag{44}$$

For impermeable boundary, the normal displacement of fluid to solid matrix is zero. Substituting Eq. (36) into Eq. (37), one obtains

$$w_{\tilde{x}} = \frac{\partial}{\partial z}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} + \alpha_1\mathrm{i}\psi)\mathrm{e}^{\mathrm{i}\gamma} + \frac{\partial}{\partial \bar{z}}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} - \alpha_1\mathrm{i}\psi)\mathrm{e}^{-\mathrm{i}\gamma} = 0 \tag{45}$$

Using Eqs. (40) and (41) and Eq. (44) to solve the permeable boundary problems and Eqs. (40) and (41) and Eq. (45) to solve the impermeable boundary problem.

5. Plane wave scatter of multiple cavities

Fig. 1 illustrates the problem of multiple cavities in saturated soil, which will be analyzed in this paper. In the steady state case, the incident plane harmonic wave can be expressed as

$$\varphi^i = \varphi_0 \exp[\mathrm{i}k_{\mathrm{f,s}}(x\cos\beta + y\sin\beta)]\mathrm{e}^{-\mathrm{i}\varpi t} \tag{46}$$

By introducing complex variables $z = x + \mathrm{i}y$, $\bar{z} = x - \mathrm{i}y$, the incident plane wave φ^i can also be written as the following formula:

$$\varphi^i = \varphi_0 \exp\left[\frac{\mathrm{i}k_{\mathrm{f,s}}}{2}(\bar{z}\mathrm{e}^{\mathrm{i}\beta} + z\mathrm{e}^{-\mathrm{i}\beta})\right]\mathrm{e}^{-\mathrm{i}\varpi t} \tag{47}$$

where superscripts i denotes the incident components of the waves; $k_{\mathrm{f,s}}$ denotes incident fast wave and slow wave, respectively; β is the angle of incident wave; φ_0 is an amplitude of the incident wave.

For one scattering wave, the general solutions of Eqs. (19) and (20) and Eq. (21) of the jth cavity may be expressed in terms of Hankel function as follows.

$$\varphi_{\mathrm{f}j}^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} a_{jn} H_n^{(1)}(k_{\mathrm{f}}|z_j|)\left(\frac{z_j}{|z_j|}\right)^n \tag{48}$$

$$\varphi_{\mathrm{s}j}^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} b_{jn} H_n^{(1)}(k_{\mathrm{s}}|z_j|)\left(\frac{z_j}{|z_j|}\right)^n \tag{49}$$

$$\psi_j^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} c_{jn} H_n^{(1)}(k_{\mathrm{t}}|z_j|)\left(\frac{z_j}{|z_j|}\right)^n \tag{50}$$

where superscripts s denotes the scatter components of the waves; $H_n^{(1)}(\cdots)$ is the Hankel function of the first kind of order n; a_{jn}, b_{jn}, c_{jn} are arbitrary functions to be determined from the boundary conditions of the jth cavities ($j = 1 \cdots m$).

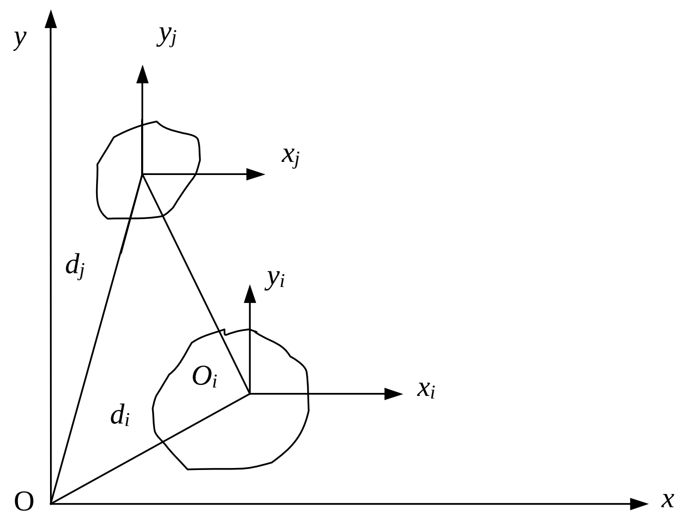

Fig. 1. Multiple cavities in saturated half space.

The total scatter waves can be expressed as

$$\varphi_{\mathrm{f}}^{\mathrm{s}}=\sum_{j=1}^{m}\sum_{n=-\infty}^{\infty}a_{jn}H_{n}^{(1)}(k_{\mathrm{f}}|z-d_{j}|)\left(\frac{z-d_{j}}{|z-d_{j}|}\right)^{n} \tag{51}$$

$$\varphi_{\mathrm{s}}^{\mathrm{s}}=\sum_{j=1}^{m}\sum_{n=-\infty}^{\infty}b_{jn}H_{n}^{(1)}(k_{\mathrm{s}}|z-d_{j}|)\left(\frac{z-d_{j}}{|z-d_{j}|}\right)^{n} \tag{52}$$

$$\psi^{\mathrm{s}}=\sum_{j=1}^{m}\sum_{n=-\infty}^{\infty}c_{jn}H_{n}^{(1)}(k_{\mathrm{t}}|z-d_{j}|)\left(\frac{z-d_{j}}{|z-d_{j}|}\right)^{n} \tag{53}$$

where d_j is the distance between the origin of jth cavities and the origin of total coordinate system. For multiple cavities of arbitrary shape in saturated soil, the total wave can be expressed as

$$\varphi_{\mathrm{f}}=\varphi_{\mathrm{f}}^{i}+\sum_{j=1}^{m}\varphi_{fj}^{\mathrm{s}}=\varphi_{\mathrm{f}}^{i}+\varphi_{\mathrm{f}}^{\mathrm{s}},\quad \varphi_{\mathrm{s}}=\varphi_{\mathrm{s}}^{i}+\sum_{j=1}^{m}\varphi_{sj}^{\mathrm{s}}=\varphi_{\mathrm{s}}^{i}+\varphi_{\mathrm{s}}^{\mathrm{s}},\quad \psi=\psi^{i}+\sum_{j=1}^{m}\psi_{j}^{\mathrm{s}}=\psi^{i}+\psi^{\mathrm{s}} \tag{54}$$

6. Stress boundary problem of multiple cavities

For the jth cavities ($j=1,2,\ldots,m$), the stresses on the boundary of cavity can be expressed as

$$\sigma_{\tilde{x}_j}-\mathrm{i}\sigma_{x\tilde{y}_j}=\alpha_{\mathrm{f}}\varphi_{\mathrm{f}}+\alpha_{\mathrm{s}}\varphi_{\mathrm{s}}+\frac{\partial^2}{\partial z_j^2}(\varphi_{\mathrm{f}}+\varphi_{\mathrm{s}}+\mathrm{i}\psi)\mathrm{e}^{2\mathrm{i}\gamma_j}=f_{1j}-\mathrm{i}f_{2j} \tag{55}$$

$$\sigma_{\tilde{x}_j}+\mathrm{i}\sigma_{x\tilde{y}_j}=\alpha_{\mathrm{f}}\varphi_{\mathrm{f}}+\alpha_{\mathrm{s}}\varphi_{\mathrm{s}}+\frac{\partial^2}{\partial \bar{z}_j^2}(\varphi_{\mathrm{f}}+\varphi_{\mathrm{s}}-\mathrm{i}\psi)\mathrm{e}^{-2\mathrm{i}\gamma_j}=f_{1j}+\mathrm{i}f_{2j} \tag{56}$$

where f_{1j} and f_{2j} are the normal and tangential total stresses of the jth cavities.

In the local coordinate system $(o_j, x_j, y_j), z_j = x_j + \mathrm{i}y_j$, $(j = 1,2,\ldots,m)$, the curvilinear equation of the jth cavities of arbitrary shape is as follows.

$$f(x_j, y_j) = 0 \tag{57}$$

The angle γ_j in arbitrary point on the boundary of cavities is

$$\gamma_j = tg^{-1}\left(\frac{\partial f/\partial y_j}{\partial f/\partial x_j}\right) \tag{58}$$

If selecting elliptic cavities as an example, the equation of the jth elliptic cavities can be expressed as

$$f(x_j, y_j)=\frac{x_j^2}{a_j^2}+\frac{y_j^2}{b_j^2}-1=0 \tag{59}$$

where a and b are long axial radius and short axial radius, respectively.

On the boundary of the jth elliptic cavities

$$x_j = r_j\cos\theta,\quad y_j = r_j\sin\theta \tag{60}$$

$$r_j=\frac{a_j}{\sqrt{\cos^2\theta+\frac{a_j^2}{b_j^2}\sin^2\theta}} \tag{61}$$

$$\gamma_j = tg^{-1}\left(\frac{a_j^2}{b_j^2} tg\theta\right) \tag{62}$$

where $\gamma = \gamma_j$, $z = z_j e^{i a_j} + d_j$, $z_j = r_j e^{i\theta}$.

From Eqs. (55) and (56), one obtains

$$\sum_{p=1}^{3}\sum_{i=1}^{m}\sum_{n=-\infty}^{n=\infty} E^1_{kpin} x_{pin} = r^1_{kj} \quad (k = 1, 2;\ j = 1, 2, \ldots, m) \tag{63}$$

where

$$E^1_{11in} = \alpha_f H_n^{(1)}(k_f|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^n + \mu k_f^2 H_{n-2}^{(1)}(k_f|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-2} e^{2i\gamma_j} \tag{64}$$

$$E^1_{12in} = \alpha_s H_n^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^n + \mu k_s^2 H_{n-2}^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-2} e^{2i\gamma_j} \tag{65}$$

$$E^1_{13in} = i\mu k_t^2 H_{n-2}^{(1)}(k_t \zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-2} e^{2i\gamma_j} \tag{66}$$

$$E^1_{21in} = \alpha_f H_n^{(1)}(k_f|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^n + \mu k_f^2 H_{n+2}^{(1)}(k_f|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+2} e^{-2i\gamma_j} \tag{67}$$

$$E^1_{22in} = \alpha_s H_n^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^n + \mu k_s^2 H_{n+2}^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+2} e^{-2i\gamma_j} \tag{68}$$

$$E^1_{23in} = -i\mu k_t^2 H_{n+2}^{(1)}(k_t|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+2} e^{-2i\gamma_j} \tag{69}$$

$$r^1_{1j} = f_{1j} - i f_{2j} - \alpha_f \varphi_f^i - \alpha_s \varphi_s^i - \mu \frac{\partial^2}{\partial z_j^2}(\varphi_f^i + \varphi_s^i + i\psi^i) e^{2i\gamma_j} \tag{70}$$

$$r^1_{2j} = f_{1j} + i f_{2j} - \alpha_f \varphi_f^i - \alpha_s \varphi_s^i - \mu \frac{\partial^2}{\partial \bar{z}_j^2}(\varphi_f^i + \varphi_s^i - i\psi^i) e^{-2i\gamma_j} \tag{71}$$

$$x_{1in} = a_{in}, \quad x_{2in} = b_{in}, \quad x_{3in} = c_{in} \tag{72}$$

where

$$\xi_{ij} = z_j e^{i a_j} + d_j - d_i \tag{73}$$

Multiplying both sides of Eq. (63) with $e^{-is\theta}$, and integrating on the interval $[-\pi, \pi]$, one obtains

$$\sum_{p=1}^{3}\sum_{i=1}^{m}\sum_{n=-\infty}^{n=\infty} E^{1s}_{kpin} x_{pin} = r^{1s}_{kj} \quad (k = 1, 2;\ j = 1, 2, \ldots, m;\ s = 0, \pm 1, \pm 2, \ldots) \tag{74}$$

where

$$E_{kpin}^{1s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} E_{kpin}^{1} \mathrm{e}^{-\mathrm{i}s\theta}\,\mathrm{d}\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{75}$$

$$r_{kj}^{1s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} r_{kj}^{1} \mathrm{e}^{-\mathrm{i}s\theta}\,\mathrm{d}\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{76}$$

For permeable boundary, the pore pressure of the *j*th cavities is zero and Eq. (44) can be written as

$$p_{\mathrm{f}j} = -A_{\mathrm{f}} k_{\mathrm{f}}^2 \varphi_{\mathrm{f}} - A_{\mathrm{s}} k_{\mathrm{s}}^2 \varphi_{\mathrm{s}} = 0 \tag{77}$$

For impermeable boundary, the normal displacement of fluid to solid matrix of the *j*th cavities is zero and Eq. (45) can be written as

$$w_{\tilde{x}_j} = \frac{\partial}{\partial z_j}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} + \alpha_1 \mathrm{i}\psi)\mathrm{e}^{\mathrm{i}\gamma_j} + \frac{\partial}{\partial \bar{z}_j}(\eta_1\varphi_{\mathrm{f}} + \eta_2\varphi_{\mathrm{s}} - \alpha_1 \mathrm{i}\psi)\mathrm{e}^{-\mathrm{i}\gamma_j} = 0 \tag{78}$$

Solving Eq. (77), one obtains

$$\sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{n=-\infty}^{\infty} E_{pin}^{2} x_{pin} = r_j^2 \quad (j = 1, 2, \ldots, m) \tag{79}$$

where

$$E_{1in}^{2} = -A_{\mathrm{f}} k_{\mathrm{f}}^2 H_n^{(1)}(k_{\mathrm{f}}|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n} \tag{80}$$

$$E_{2in}^{2} = -A_{\mathrm{s}} k_{\mathrm{s}}^2 H_n^{(1)}(k_{\mathrm{s}}|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n} \tag{81}$$

$$r_j^2 = A_{\mathrm{f}} k_{\mathrm{f}}^2 \varphi_{\mathrm{f}}^i + A_{\mathrm{s}} k_{\mathrm{s}}^2 \varphi_{\mathrm{s}}^i \tag{82}$$

Multiplying both sides of Eq. (79) with $\mathrm{e}^{-\mathrm{i}s\theta}$, and integrating on the interval $[-\pi, \pi]$, one obtains

$$\sum_{p=1}^{2}\sum_{i=1}^{m}\sum_{n=-\infty}^{\infty} E_{pin}^{2s} x_{pin} = r_j^{2s} \quad (j = 1, 2, \ldots, m \;\; s = 0, \pm 1, \pm 2, \ldots) \tag{83}$$

$$E_{pin}^{2s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} E_{pin}^{2} \mathrm{e}^{-\mathrm{i}s\theta}\,\mathrm{d}\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{84}$$

$$r_j^{2s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} r_j^{2} \mathrm{e}^{-\mathrm{i}s\theta}\,\mathrm{d}\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{85}$$

From Eq. (78), there is

$$\sum_{p=1}^{3}\sum_{i=1}^{m}\sum_{n=-\infty}^{\infty} E_{pin}^{3} x_{pin} = r_j^3 \quad (j = 1, 2, \ldots, m) \tag{86}$$

where

$$E_{1in}^{3} = \frac{\eta_1 k_{\mathrm{f}}}{2} H_{n-1}^{(1)}(k_{\mathrm{f}}|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-1} \mathrm{e}^{\mathrm{i}\gamma_j} - \frac{\eta_1 k_{\mathrm{f}}}{2} H_{n+1}^{(1)}(k_{\mathrm{f}}|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+1} \mathrm{e}^{-\mathrm{i}\gamma_j} \tag{87}$$

$$E_{2in}^3 = \frac{\eta_2 k_s}{2} H_{n-1}^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-1} e^{i\gamma_j} - \frac{\eta_2 k_s}{2} H_{n+1}^{(1)}(k_s|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+1} e^{-i\gamma_j} \tag{88}$$

$$E_{3in}^3 = \frac{i\alpha_1 k_t}{2} H_{n-1}^{(1)}(k_t|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n-1} e^{i\gamma_j} + \frac{i\alpha_1 k_t}{2} H_{n+1}^{(1)}(k_t|\zeta_{ij}|)\left(\frac{\zeta_{ij}}{|\zeta_{ij}|}\right)^{n+1} e^{-i\gamma_j} \tag{89}$$

$$r_j^3 = -\frac{\partial}{\partial z_j}(\eta_1 \varphi_f^i + \eta_2 \varphi_s^i + \alpha_1 i\psi^i)e^{i\gamma_j} - \frac{\partial}{\partial \bar{z}_j}(\eta_1 \varphi_f^i + \eta_2 \varphi_s^i - \alpha_1 i\psi^i)e^{-i\gamma_j} \tag{90}$$

Multiplying both sides of Eq. (86) with $e^{-is\theta}$, and integrating on the interval $[-\pi, \pi]$, one obtains

$$\sum_{p=1}^{3}\sum_{i=1}^{m}\sum_{n=-\infty}^{\infty} E_{pin}^{3s} x_{pin} = r_j^{3s} \quad (j = 1, 2, \ldots, m;\ s = 0, \pm 1, \pm 2, \ldots) \tag{91}$$

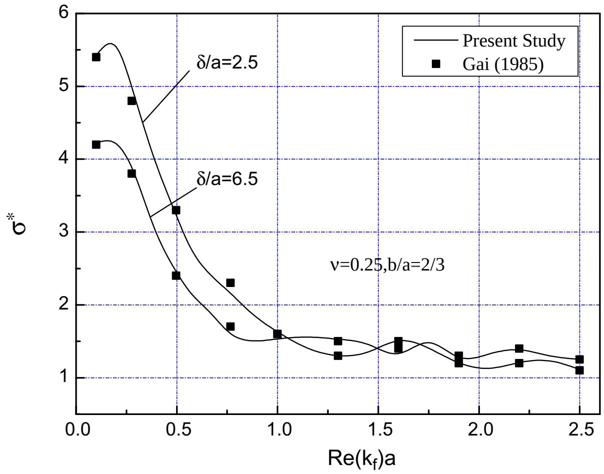

Fig. 2. Comparison of calculated between present result and Gai (1985).

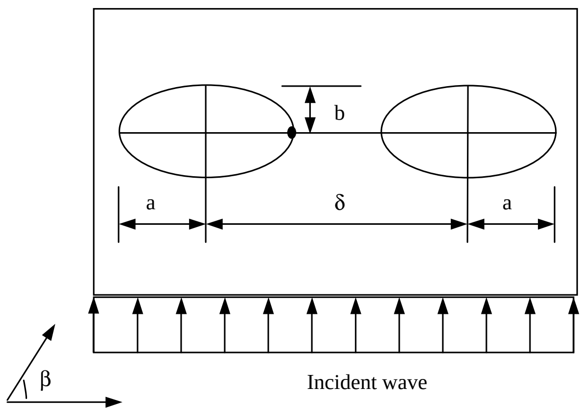

Fig. 3. Inclusion of pressure wave to two same elliptic cavities in saturated soil.

$$E_{pin}^{3s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} E_{pin}^{3} e^{-is\theta}\, d\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{92}$$

$$r_j^{3s} = \frac{1}{2\pi}\int_{-\pi}^{\pi} r_j^{3} e^{-is\theta}\, d\theta \quad (s = 0, \pm 1, \pm 2, \ldots) \tag{93}$$

Eqs. (74), (83) and (91) form a set of infinite algebraic equations for determining the functions a_{jn}, b_{jn}, and c_{jn}. Eqs. (74) and (83) are selected to solve the problem of permeable boundary and Eqs. (74) and (91) are selected to solve the problem of impermeable boundary.

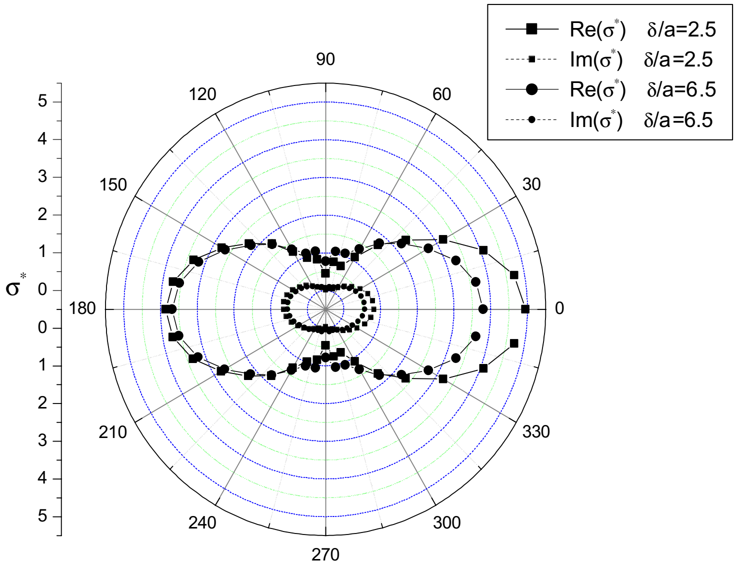

Fig. 4. Distribution of stresses around boundary of left cavities in permeable condition ($\mathrm{Re}(k_f)a = 0.1$).

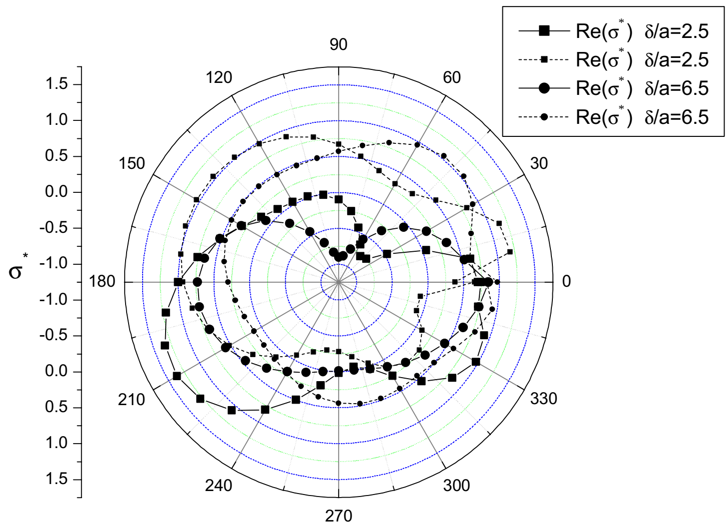

Fig. 5. Distribution of stresses around boundary of left cavities in permeable condition ($\mathrm{Re}(k_f)a = 1.0$).

7. Numerical results

In engineering practice, the dynamic stress concentration factor is most significant aspects in the study of wave propagation in the saturated soil. In this study, dynamic stress concentration factors will be calculated in the case of incident plane wave is fast wave. The dynamic stress concentration factor is defined as the ratio of tangential effective stress to the maximum amplitude of the incident effective stress at the same point

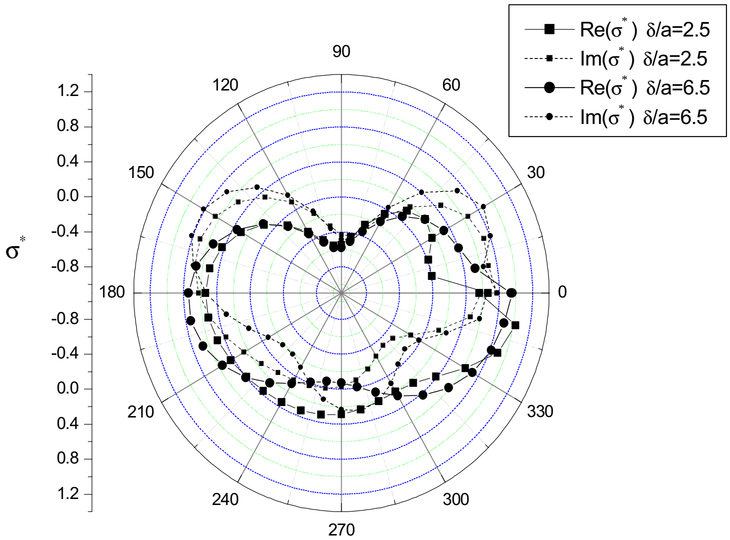

Fig. 6. Distribution of stresses around boundary of left cavities in permeable condition ($\mathrm{Re}(k_f)a = 2.0$).

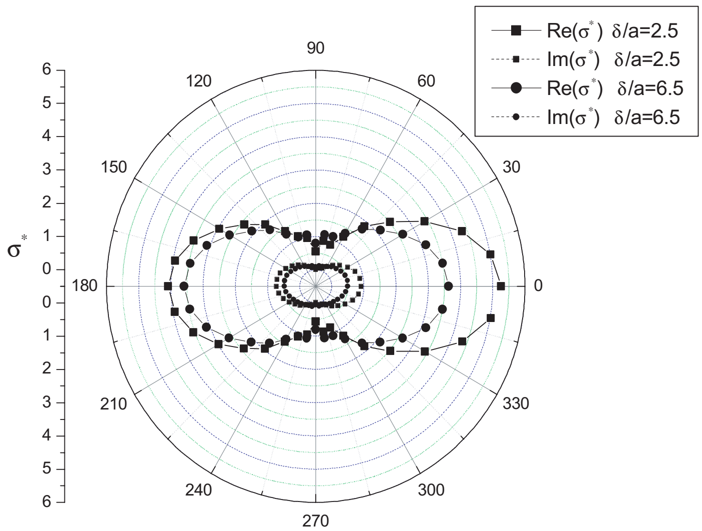

Fig. 7. Distribution of stresses around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_f)a = 0.1$).

$$\sigma^* = \frac{\sigma_{\bar{y}}}{\sigma_0} \tag{94}$$

where

$$\sigma_0 = \mathrm{Re}[-(\lambda + 2\mu)k_f^2\varphi_0] \tag{95}$$

For the case of impermeable condition, the pore pressure concentration factor is defined as the ratio of the pore pressure on the boundary of cavity to the maximum amplitude of pore pressure at the same point

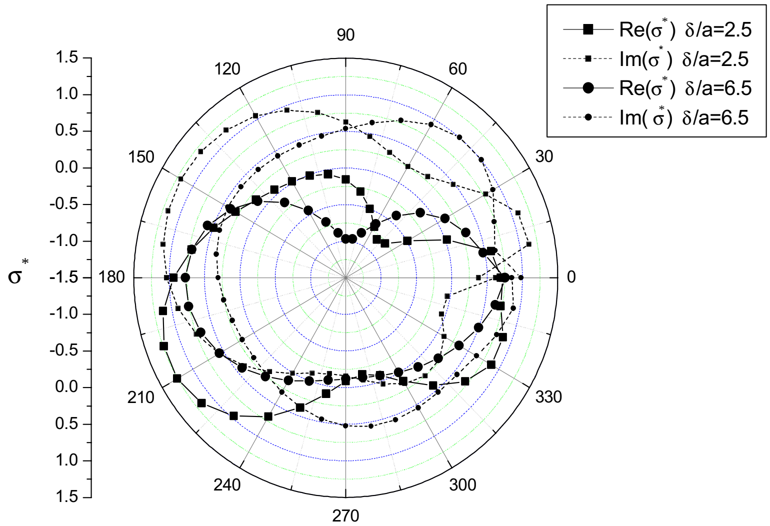

Fig. 8. Distribution of stresses around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_f)a = 1.0$).

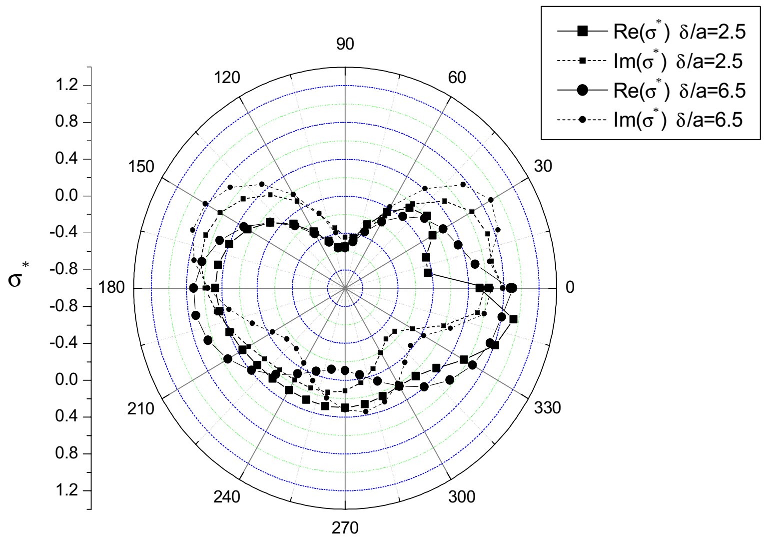

Fig. 9. Distribution of stresses around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_f)a = 2.0$).

$$p_{\mathrm{f}}^{*} = \frac{p_{\mathrm{f}}}{p_{f0}} \tag{96}$$

where

$$p_{f0} = \mathrm{Re}(-A_{\mathrm{f}} k_{\mathrm{f}}^{2} \varphi_{0}) \tag{97}$$

In order to confirm the accuracy of the present method, it is assumed that the saturated soil is the ideal elastic medium. Fig. 2 shows a comparison of proposed solution with that reported by Gai (1985). As illustrated in Fig. 2, it is obviously that the accuracy of proposed solution is higher.

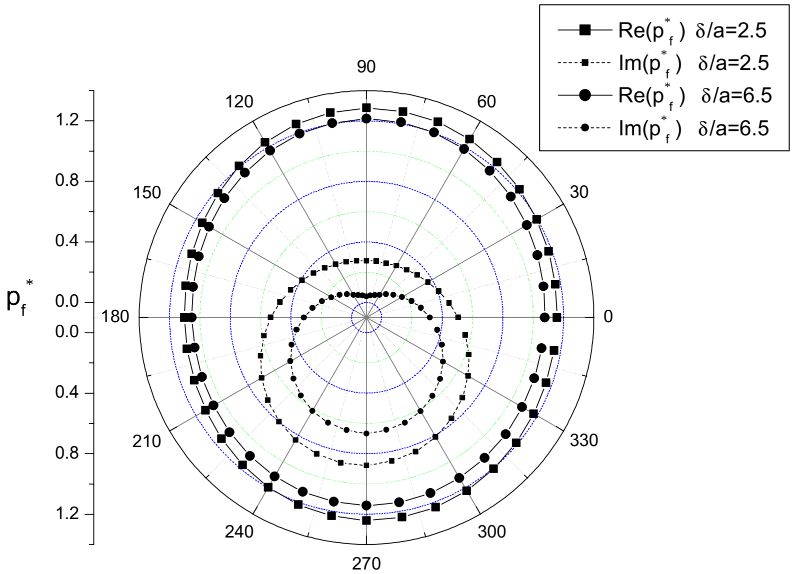

Fig. 10. Distribution of pore pressures around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_{\mathrm{f}})a = 0.1$).

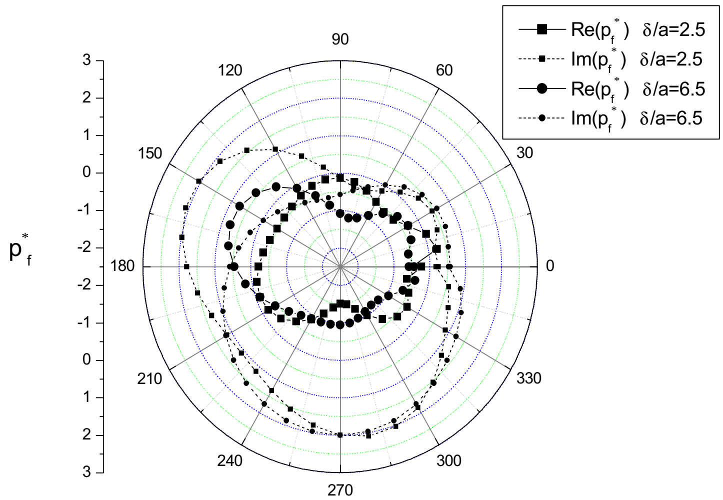

Fig. 11. Distribution of pore pressures around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_{\mathrm{f}})a = 1.0$).

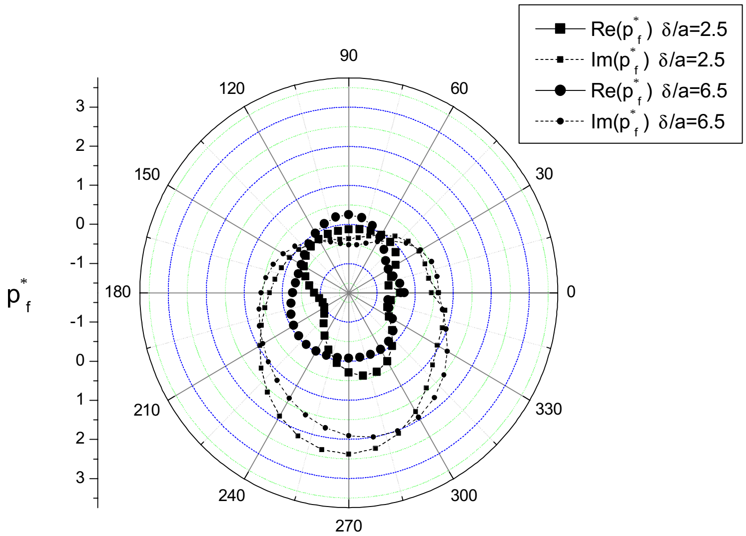

Fig. 12. Distribution of pore pressures around boundary of left cavities in impermeable condition ($\mathrm{Re}(k_f)a = 2.0$).

In the following analyses, the problem of two elliptic cavities as illustrated in Fig. 3 is considered. The two elliptic cavities are embedded in the saturated soil. The parameters are: $\beta = 90°$; $\rho_s = 2500\ \mathrm{kg/m^3}$; $\rho_f = 1000\ \mathrm{kg/m^3}$; $n = 0.3$; $\mu = 1.0 \times 10^7$ Pa; passion ratio $v = 0.4$; $\alpha = 0.999$; $M = 1.0 \times 10^8$ Pa; $\eta = 1.0 \times 10^{-2}$ Pa s; $k = 1.0 \times 10^{-7}\ \mathrm{m^2}$; $b/a = 0.75$; $\delta/a = 2.5$, 6.5. Figs. 4–6 show the distributions of dynamic stress concentration factors along the boundary of left cavity under the cases of permeable condition at dimensionless fast wave number $Re(k_f)a = 0.1$, 1.0, 2.0. Figs. 7–12 show the distributions of dynamic stress concentration factors and pore pressure concentration factors along the boundary of left cavity in impermeable condition at $Re(k_f)a = 0.1$, 1.0, 2.0.

8. Conclusions

The complex variable method has been employed to solve the problem of scattering of plane harmonic wave by considering two elliptic cavities embedded in saturated soil. The result shows that the influence of dimensionless distance δ/a and dimensional wave number $\mathrm{Re}(k_f)a$ are more visible in the dynamic stress concentration factors. When dimensionless wave number $Re(k_f)a = 0.1$, the dynamic stresses at $\delta/a = 2.5$ are greater than those at $\delta/a = 6.5$. The dynamic stress concentration factors in permeable condition are smaller than those in impermeable condition. The dynamic stresses and pore pressures decrease with the incident wave number increasing. For the case of impermeable boundary condition, the pore pressures are less than dynamic stresses.

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References

Biot, M.A., 1941. General theory of three dimensional consolidation. Journal of Applied Physics 12, 55–164.

Biot, M.A., 1956. Theory of propagation of elastic waves in a fluid–saturated porous solid, I, low frequency rang. Journal of the Acoustical Society of America 28 (2), 168–178.

Biot, M.A., 1962. Mechanics of deformation and acoustic propagation in porous media. Journal of Applied Physics 33 (4), 1482–1498.

Datta, S.K., Wong, K.C., Shah, A.H., 1984. Dynamic stress and displacement around cylindrical cavities of arbitrary shapes. Journal of Applied Mechanics, ASME 51, 798–803.

Degrande, G., de Roeck, G., Van Broeck, P., Semulders, D., 1998. Wave propagation in layered dry, saturated and unsaturated poroelastic media. International Journal of Solid and Structures 35, 4753–4778.

Gai, B.Z., 1985. The problem of the diffraction of the elastic wave by a group of elliptic cavities. Acta Mechanica Solid Sinica 3, 353–359.

Gamer, U., 1977. Dynamic stress concentration in an elastic half space with a semi-circular cavity excited by SH waves. International Journal of Solid and Structures 13, 675–681.

Kattis, S.E., Beskos, D.E., Cheng, A.H.D., 2003. 2D dynamic response of unlined and lined tunnels in poroelastic soil to harmonic body waves. Earthquake Engineering and Structural Dynamics 32 (1), 97–110.

Liu, D.K., Gai, B.Z., Tao, G.Y., 1981. Discussion of the dynamic stress concentration nearby cavity. Acta Mechanica Sinica 12, 65–77.

Mei, C.C., Foda, M.A., 1981. Wave-induced responses in a fluid-filled poro-elastic solid with a free surface—A boundary layer theory. Geophysics Journal of the Royal Astronomical Society 66, 597–631.

Mei, C.C., Si, B.I., Cai, D.Y., 1984. Scattering of simple harmonic waves by a circular cavity in a fluid-filled treated poro-elastic medium. Wave Motion 6 (3), 265–278.

Norris, A.N., 1985. Radiation from a point source and scattering theory in a fluid–saturated porous solid. Journal of the Acoustical Society of America 77 (6), 2012–2023.

Sancar, S., Pao, Y.H., 1981. Spectral analysis of elastic pulses back scattered from two cylindrical cavities in a solid. Journal of the Acoustical Society of America 69 (6), 1591–1596.

Zimmerman, C., Stern, M., 1993. Boundary element solution of 3-D wave scatter problems in a poroelastic medium. Engineering Analysis with Boundary Elements 12, 223–240.

Zitron, N.R., 1967. Multiple scattering of elastic waves by two arbitrary cylinders. Journal of the Acoustical Society of America 42 (3), 620–624.